## **PF - Assignment # 1: Challenges**

**Challenge 1.1**  
1. You have 2 drums of capacity

* 8 litres
* 5 litres

And a resource of Oil, Can you use these two drums to give 1 litre, 2-litre, 3 litre, 4 litre, and 6 litres and 7 litres to any customer. Note that there is no mark of litres on the drums.

2. Now you have to measure again the same question: But you have a 9-litres and 6 litres drum. Can you solve that same problem, if yes how? If not, why?

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**Directions to look at of Problem 1.1**

For example you can easily measure 3 litres in the following way; take an 8 litre drum, let us call it A and pour it into a 5 litre drum, let us call it B, until B gets filled. Now the remaining litres in A will be 3. Similarly, repeat the same procedure (you can throw the filled drum again into the resource of Oil).

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**Challenge 1.2**

On your first day at university, the teacher suggested that it would be a good idea for each student to meet every other student in the class. The teacher said, "When you meet, please shake hands and introduce yourself by name." If there were 20 students in the class, how many total handshakes happened? What about if there are N students can you tell in terms of N what is the total count of shake-hands.

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**Directions to look at of Problem 1.2**

One student will hand-shake with another student only once. Let say if we have 3 students A, B ,C then A student will hand-shake with B and C and B will hand-shake with C only because B has already have a hand-shake with A. Now C will not hand -shake with A and B because C has already have a hand-shake with A and B.

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**Challenge 1.3**

If you have an 8 x 8 chessboard, how many squares (within that chessboard) would there be in total? (64 is not the answer).

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**Directions to look at of Problem 1.3**

You must calculate each possible combination of squares on board i.e., 1x1, 2x2,...,8x8 have how many square boxes and then sum up these # of boxes.

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**Challenge 1.4**

You are given 2 balls made of clay. You have access to a 100-story building. Balls can be very hard or very fragile means it may break if dropped from the first floor or may not even break if dropped from the 100th floor. Both balls are identical. You need to figure out that on the highest floor of a 100-story building a ball can be dropped without breaking. The question is how many minimum drops do you need to make to find that story. You are allowed to break 2 balls in the process.

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**Directions to look at Problem 1.4**

First, try to solve the problem with only one ball. And to solve with one ball you can’t solve that with binary solutions like if you drop that from the 50th floor and it breaks then you are bound. So, for one ball you can only solve that by dropping the ball from the 1st floor than from the 2nd and so on. But in that case, we have a lot of steps to solve. Like if You have an N floor then you have N-1 tries.

Now as you have two balls to solve that problem, you can first drop that from the 50th floor and if it breaks then you can use the same solution which we used in case of one ball from the 1-49th floor and if it doesn’t break then you drop the second ball from 51 to 100th floor. Now see that, to figure out the solution with one ball we have done 99 tries (worst case). But in that case with two balls we will just try 49 times to solve the solution in the worst case. **Now the question is, can you do better than that?**

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**Challenge 1.5**

Divide RS: 277 (in whole $ increments) into a number of bags so that I can ask for any amount between RS: 1 and RS: 277, and you can give me the proper amount by giving me a certain number of these bags without opening them. What is the minimum number of bags you will require?

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**Directions to look at of Problem 1.5**

You must derive a sequence like if you make a bag having RS: 1 and RS: 3 then there is no need to make a bag of RS: 4 again because 4 can be paid using RS: 1 and RS: 3 bag combined. Similarly try different possibilities to figure out what should be the minimum sequence to have so that every amount from 1 to 277 can be paid with the subset of the set of bags you will make.   
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**Challenge 1.6**

A corporate businessman has two cubes on his office desk. Every day he arranges both cubes so that the front faces show the current day of the month. What numbers are on the faces of the cubes to allow this?

Note: You can't represent the day "7" with a single cube with a side that says 7 on it and the other cube hidden. You have to use both cubes all the time. So the 7th day should be represented like "07".

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**Directions to look at Problem 1.6**

You must detect that numbers which can represent every day of the month and a number on one cube can be on another cube but repetition of numbers on the same cube is not allowed. One cube can have a series like 0,1,2,3,4,5. Now you have to detect the numbers on the second cube.

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**Challenge 1.7**

A pot contains 75 white beans and 150 black ones. Next to the pot is a large pile of black beans.

A somewhat crazy cook removes the beans from the pot, one at a time, according to the following strange rule: He removes two beans from the pot at random.

* If at least one of the beans is black, he places it on the bean-pile and drops the other bean, no matter what colour, back in the pot.
* If both beans are white, on the other hand, he discards both of them and removes one black bean from the pile and drops it in the pot.

Notice at each turn of this procedure, the pot has one less bean in it. Eventually, just one bean is left in the pot. What colour is it?

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**Directions to look at of Problem 1.7**

1. black black beans: (-1) black bean
2. Black white beans: (-1) black bean
3. White white beans: (-2) white (+)1 black

This is the pattern of beans are added back to the pot or removed from it. Now determine at end which bean will be in the pot according to these rules. *Try solving with a small example with 5 white beans and 10 black beans with random choices of pairs*.

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**Challenge 1.9**

You have an infinite plane and lines, lines can intersect each other to make regions, e.g. 1 line will produce 2 regions, 2 lines 4. Can you tell with argument at maximum how many regions will be there if you have n lines? And why? (Any answer without reason will be unacceptable).

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**Directions to look at of Problem 1.9**

From above we can determine that 1 line produces 2 (i.e., 1x2) and 2 lines produce 4 (i.e., 2x2), with three lines you can have at max 7 regions. Now determine a generalisation for N lines what my number of regions will be. Look very carefully at what happens with the introduction of a new line and how it will make the maximum number of regions? Use the help of drawing.

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**Challenge 1.10**

* A duck is pursued by a fox, escapes to the centre of a perfectly circular pond. The fox cannot swim and the duck cannot take flight from the water. The fox is 4 times faster than the duck. Assuming the fox and duck pursue optimal strategy, is it possible for the duck to reach the edge of the pond and fly away without being eaten? If so, how? (You can find its answer on the Internet very easily, you can use the Internet but I would say first try for it and then go on finding it) [2]
* On what parameters (relation between their speeds) fox will always be able to eat it, and on what factors duck will always escape.

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**Directions to look at of Problem 1.10**

Use the freedom of the movement of the duck inside the pond such that the duck can have the prior distance covered already so that when the duck decides to move in a certain direction of reaching to the corner such that when it reaches and puts a foot on the ground fox is still lagging in the chase.

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**Challenge 1.11**

You have two strings whose only known property is that when you light one either end of either string it takes exactly one hour to burn. The rate at which the strings will burn is completely random and each string is different. How do you measure 1.5 hours and 45 minutes?

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**Directions to look at of Problem 1.11**

In order to measure 1.5 hours you firstly burn a string from one side and when it burns completely 1 hour is passed then simultaneously burn second string from both sides and when it is burned completely 1.5 hours are gone. Now use a similar strategy to calculate 45 minutes.

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**Challenge 1.12**

There are four numbers: 1, 2, 5, 10 that you have to carry in a boat across a river. Every time you may not carry more than two numbers , and the Cost of travelling is associated with the largest of two numbers. One number has to be in the boat during the return journey, which also defines the Cost for this trip. What is the minimum total Cost to carry all numbers from one bank to another?

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**Directions to look at Problem 1.12**

If we take the number 1 and 2 to cross the river or take back the number 1, then it will cost us 2+1 = 3.

Then we take number 5 and 10 and return back with 5 (10+5 = 15). And at last we take 5 and 1 across the river that makes it total cost of 23. **Can we do it in a better way?**

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**Challenge 1.13 (Exponential Search)**

Let us play a game. This is a One-Dimensional game in which we have many people/lives(**N** people) in a single row. Accidently these people/lives are affected by a deadly virus. This virus spreads to people one by one in a specific order and maybe when we start the game then there are already many people/lives affected by the virus but fortunately, Dr. Sarim made the antidote for that virus. As we know, viruses affect people continuously. So, when we check any life for a virus that affects that life or not, then it will take a little bit of time but during that time the virus will affect another life who will be next to the previous affected life. We can check and give an antidote to people/life one by one, meaning only one life will be checked and get an antidote at a time. So, we need to save the maximum number of people from the virus. How many minimum people will be affected by the virus during finding the last affected by the virus?

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**Direction to look at Problem 1.13**

Suppose there are 100 people/lives in this game and we need to save maximum people from the deadly virus. First, we will check mid-life, which is the 50th life, so we will check whether the 50th life is affected or not. If affected then we will check for mid of 50 & 100 which is 75th life. But if the 50th life is not affected then we give him an antidote and check for mid of 0 & 50 which is 25. So, by that process, we can save 94 people/life out of 100 people/life. But what about we have **N** people/life. You have to save as many people/lives as possible from the deadly virus.

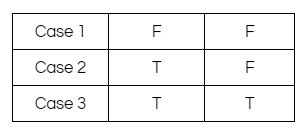
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**The Open Challenge**

You wake up after your Titanic sank, and you find yourself on the shore of the sea, an island. Besides everything that an island has (its natural beauty), there are tribesmen there too. You come to know that every tribesman of this island is either faker(he may or may not lie) or trustworthy(always tell the truth). You can ask them questions like who is a faker. And who is trustworthy? There are a total of 100 people on this island. And you can ask as many questions as you want. The only information known is that the number of Trustworthiness is greater than fakers. Any answer having greater than 500 questions is not worthy enough to be considered. Figure out who is a faker and who is a trustworthy person?

| **The Geometric Series**  Geometric Series is one of the most essential and exists in abundance in nature and also in computational related problems in CS. It looks like the following:  **1+ b + b2 + b3 + b4 + … + N/b2 + N/b1 + N** or also written as **N + N/b + N/b2 + … + b4+b3+b2+b+1**  The left side is known as increasing geometric series and the right side is decreasing geometric series both ending and starting with N.  It is not hard to prove that that the summation of the above series is bounded by a constant time the largest term i.e. some C x N. See Wikipedia for the proof [**https://en.wikipedia.org/wiki/Geometric\_series**](https://en.wikipedia.org/wiki/Geometric_series)  Examples of the above series could be the following series with b = 2. **1+2+4+8+16+...+ N/4 + N/2 + N** also written as **⇒ N + N/2 + N/4 + N/8 + ….+ 16 + 8 + 4 + 2 + 1**  It is not hard to prove that the summation of the above series  (2 versions increasing as well as the decreasing geometric series is always less than 2N. The proof is quite simple and very simple to figure out what it means. Imagine that the largest term N is actually one big Square S (the grey one) taking two big squares now the rest of the summation can never together sum up and fill in the 2nd square G(the green one). Very simply start shading the summed up region. If N is a complete square S then N/2 will be half of the square G. Similarly N/4 will be a quarter of the square G and so on. Hence you will never be able to fill in the 2nd square G. That completes the proof that the summation of the geometric series with increasing or decreasing factor b=2 is always bounded by <2N. |
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**Directions to look at of Problem**

You have to find at least one trustworthy person by whom you can ask who is a trustworthy or liar. The strategy is inspired by the beautiful Geometric series. One idea could have been to take one person out and ask from everyone “Is he the trustworthy one?” if the majority say yes then we have found the one trustworthy and we may ask him about everyone. But the issue is you could be unlucky in that case you have to choose the 2nd person and repeat that would be a very bad solution. It will work but in that case you may have to almost select 49 bad choices of fakers.   
Now the idea is make pair of 2 each (resulting 50 pairs i.e. N/2 pairs) and just ask those pairs about each other Here could be the possible answers (F for faker, T for Trustworthy).

In all those cases where cases 1 and 2 arise we reject the pairs. In case 3 we choose one representative and ask the other person in the pair to sit down. With this one process greater than half of the people have been pruned and you should prove that “the Trustworthy-men in majority” condition will still be valid (you should prove that?). With this one step we have asked 100 questions but the searching space have reduced to half, now we repeat the same process of shrinking and on the worst case the number of questions will look like the following series

100 + 50 + 25 + … + 1 that will always be less than 200 questions. Hence we have found one Trustworthy in around 200 questions.

Be careful with this procedure. It may be possible that there is an odd number of people remaining on a certain step. What should we do in that case? We have to think about that. The hint is in incorporating that issue we might have to do some extra steps which might take the number of questions close to 300.

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